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# Astrophysical Constraints on Large Extra Dimensions

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## Abstract

In the Kaluza-Klein (KK) scenario with $n$ large extra dimensions where gravity propagates in the $4+n$ dimensional bulk of spacetime while gauge and matter fields are confined to a four dimensional subspace, the light graviton KK modes can be produced in the Sun, red giants and supernovae. We study the energy-loss rates through photon-photon annihilation, electron-positron annihilation, gravi-Compton-Primakoff scattering, gravi-bremsstrahlung and nucleon-nucleon bremsstrahlung, and derive lower limits to the string scale $M_S$. The most stringent lower limit obtained from SN1987A leads to $M_S >$ $30-130$ TeV ($2.1-9.3$ TeV) for the case of two (three) large extra dimensions.

## I. INTRODUCTION

Recently there has been revived interest in physics of extra spatial dimensions. Compact spatial dimensions with inverse radius at the order of the grand unified scale $\sim 10^{16-17}$ GeV are familiar ingredients in string compactifications and have been studied extensively since the mid-80's [1]. However, recent developments in string duality suggest that it is possible to have a much lower string or compactification scale [2]. In particular, it is conceivable to set the scale at the order of a TeV, corresponding to a weak-scale string theory [3]. Such a low string scale has the phenomenological attraction of a lighter and experimentally accessible string state spectrum. Furthermore if the large Planck mass is attributed to the existence of $n$ extra dimensions, then the sizes of these extra dimensions ($R$) can be in the range of 1 fm to 1 mm for $n = 6$ to 2 [4]. The case of one large extra dimension implies modifications of Newton's law in the range of earth-sun distance and is therefore excluded. For $n = 2$ with $R \sim \mathcal{O}(1 \text{ mm})$, it might be probed with laboratory gravitational experiments [5].

We consider the scenario that only gravity propagates in the extra dimensions, while the Standard Model (SM) fields and interactions are "confined" to a four-dimensional subspace. In this scenario, the effect of large extra dimensions arises only from interactions involving the Kaluza-Klein (KK) excitations of the gravitons from compactification. At an energy scale much lower than the string scale, one can construct an effective theory of KK gravitons interacting with the standard model fields [6]. Each graviton KK state couples to the SM field with the gravitational strength according to

$$\mathcal{L} \;=\; -\frac{\kappa}{2} \sum_{\vec{n}} \int d^4x \; h^{\mu\nu,\vec{n}} T_{\mu\nu} \;, \tag{1}$$

where $\kappa = \sqrt{16\pi G_N}$, and the summation is over all KK states labeled by the level $\vec{n}$. $T_{\mu\nu}$ is the energy-momentum tensor of the SM and $h^{\mu\nu,\vec{n}}$ the KK state with mass

$$m_{\vec{n}}^2 = \vec{n}^2/R^2 \;.$$

We note that Eq. (1) is of the same form as that for massless graviton-matter coupling. Since for large $R$ the KK gravitons are very light, they may be copiously produced in high energy processes. For real emission of the KK gravitons from a SM field, the total cross-section can be written as

$$\sigma_{\rm tot} \;=\; \kappa^2 \sum_{\vec{n}} \sigma(\vec{n}) \;, \tag{2}$$

where the dependence on the gravitational coupling is factored out. Because the mass separation of adjacent KK states, $\mathcal{O}(1/R)$, is usually much smaller than typical energies in a physical process, we can approximate the summation by an integration. Identifying the relation between the Planck mass in 4-dimension ($M_{\rm Pl}$) and that in $(4+n)$-dimension ($M_S$) according to

$$\Omega_n M_{\rm Pl}^{-2} R^n \;=\; M_S^{-(n+2)} \;, \tag{3}$$

where $\Omega_n$ is the $n$-dimensional spherical volume, one can immediately infer that $\sigma_{\rm tot}$ has an $M_S^{-(n+2)}$ dependence. Thus the large degeneracy of the KK states compensates for the

weakness of a single KK interaction. The associated rich collider phenomenology has been the topic of many recent studies [6–9].

In this paper, we study astrophysical consequences of this scenario in the effects of KK graviton emission in hot stars such as the Sun, red giants and supernova SN1987A. As in the classic example of an invisible axion [10], excessive energy losses in the stars can alter the stellar evolution and severe constraints can thereby be placed on any weakly-interacting light particle like the KK graviton. We first compute in Sec. 2 the energy-loss rate for various processes involving emission of the KK gravitons ($G_{KK}$), which include

(a) $\gamma\gamma \to G_{KK}$, Photon-photon annihilation;

(b) $e^- e^+ \to G_{KK}$, Electron-positron annihilation;

(c) $e^- \gamma \to e^- \; G_{KK}$, Gravi-Compton-Primakoff scattering;

(d) $e^-(Ze) \to e^-(Ze) \; G_{KK}$, Gravi-bremsstrahlung in a static electric field;

(e) $NN \to NN \; G_{KK}$, Nucleon-nucleon bremsstrahlung.

We then use these formulae to derive lower limits on the string scale $M_S$ in Sec. 3. We summarize our results and conclude in Sec. 4.

Many of the processes listed above were considered first in Ref. [7], with rate estimates based only on dimensional analysis. When our calculation was in progress, another related work appeared [11], in which the nucleon-nucleon bremsstrahlung process was studied in detail. Our results for this process are consistent with their calculations.

## II. STAR ENERGY-LOSS VIA KK GRAVITONS

Weakly-interacting light particles may result in energy losses for hot stellar objects such as the Sun, red giants and SN1987A, with the invisible axion as the classic example [10,12,13]. We here study the energy loss due to the escaping KK gravitons. An important difference from the axion case is that the KK graviton and matter interactions are of gravitational strength, so the KK states never become thermalized and always freely escape. In this Section, we calculate the volume energy-loss rates (emissivities) for various processes via KK graviton emission. The energy-loss rates have a high-power dependence on the string scale, namely of $M_S^{-(n+2)}$, and corrections to our approximate calculations would not significantly alter the lower limits on $M_S$ that we obtain.

### A. Photon-photon annihilation to $KK$ gravitons

Photons are abundant in hot stars. We first consider photon-photon annihilation to a KK graviton. Unlike the invisible axion, the KK gravitons couple to photons at the tree-level, as shown in Fig. 1(a). Using the Feynman rules derived in Ref. [6], the spin-averaged total cross-section for this process is easily found to be

$$\sigma_{\gamma\gamma\to G_{KK}}(s, m_{\vec{n}}) \;=\; \frac{\pi\kappa^2\sqrt{s}}{16}\delta(m_{\vec{n}} - \sqrt{s}) \;, \tag{4}$$

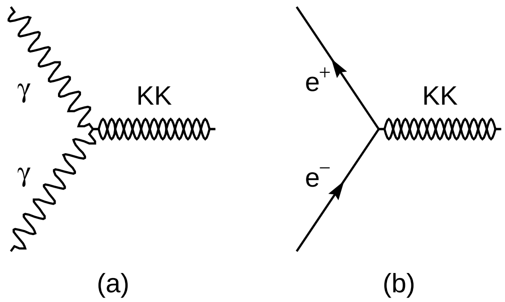


FIG. 1. Feynman diagrams for (a) photon-photon and (b) electron-positron annihilation into a KK graviton. We represent KK gravitons by double-sinusoidal curves.

where $s$ is the center of mass energy, and $m_{\vec{n}}$ the mass of the KK state at level $\vec{n}$.

The volume emissivity of a hot star with a temperature $T$ through this process is obtained by thermal-averaging over the Bose-Einstein distribution [1]

$$Q_\gamma = \int \frac{2d^3\vec{k}_1}{(2\pi)^3}\frac{1}{e^{\omega_1/T}-1}\int \frac{2d^3\vec{k}_2}{(2\pi)^3}\frac{1}{e^{\omega_2/T}-1}\frac{s(\omega_1+\omega_2)}{2\omega_1\omega_2}\sum_{\vec{n}}\sigma_{\gamma\gamma\to G_{KK}}(s,m_{\vec{n}}), \tag{5}$$

where the summation is over all KK states, and the squared center of mass energy $s$ is related to the photon energies and the angle between the two photon momenta $\theta_{\gamma\gamma}$ as follows:

$$s = 2\omega_1\omega_2(1-\cos\theta_{\gamma\gamma}) \ . \tag{6}$$

After carrying out the integrals and the summation over KK states, we find

$$Q_\gamma = \frac{2^{n+3}\Gamma(\frac{n}{2}+3)\Gamma(\frac{n}{2}+4)\zeta(\frac{n}{2}+3)\zeta(\frac{n}{2}+4)}{(n+4)\pi^2}\frac{T^{n+7}}{M_S^{n+2}} \ , \tag{7}$$

where we have used Eq. (3). Numerically, these Riemann zeta-functions are close to 1. In this calculation, we have neglected the plasma effect, through which the photons can have different energy dispersion relations from those of free particles [12].

### B. Electron-positron annihilation to $KK$ gravitons

In supernovae, the core temperature ($T_{\rm SN}$) is high enough for pair-creation of electrons and positrons, which subsequently annihilate to KK gravitons, as depicted in Fig. 1(b), with a total cross-section (neglecting the electron mass since $m_e \ll T_{\rm SN}$) given by

$$\sigma_{e^-e^+\to G_{KK}}(s,m_{\vec{n}}) = \frac{\pi\kappa^2\sqrt{s}}{64}\delta(m_{\vec{n}}-\sqrt{s}) \ . \tag{8}$$

The volume emissivity is obtained by thermal-averaging over the Fermi-Dirac distribution

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[1] This expression is similar to that of the energy loss rate via $\gamma\gamma\to\nu\bar{\nu}$ [14].

$$Q_e = \int \frac{2d^3\vec{k}_1}{(2\pi)^3}\frac{1}{e^{(E_1-\mu_e)/T}+1}\int \frac{2d^3\vec{k}_2}{(2\pi)^3}\frac{1}{e^{(E_2+\mu_e)/T}+1}\frac{s(E_1+E_2)}{2E_1E_2}\sum_{\vec{n}}\sigma_{e^-e^+\to G_{KK}}(s,m_{\vec{n}})$$
$$= \frac{2^n I_e(n)}{(n+4)\pi^2}\frac{T^{n+7}}{M_S^{n+2}}\ , \qquad (9)$$

where $\mu_e$ and $-\mu_e$ are the chemical potentials for electrons and positrons; $\mu_e \simeq (3\pi^2 n_e)^{1/3} \simeq$ 345 MeV with the electron density $n_e \simeq 1.8\times 10^{38}$ cm$^{-3}$ at the supernova core. The integral factor is

$$I_e(n) \ = \ \int_0^\infty dx\ dy\ \frac{(xy)^{n/2+2}(x+y)}{(e^{x-\mu_e/T}+1)(e^{y+\mu_e/T}+1)}\ . \qquad (10)$$

Numerically, the value of this integral ranges from 0.08 to 86 ($n=2$) and from 0.62 to 450 ($n=3$) for $T_{\rm SN}$ from 20 MeV to 60 MeV. We note that the energy-loss rate formula Eq. (9) can also be applied to neutrino-antineutrino annihilation at the supernova neutrino sphere.

### C. Gravi-Compton-Primakoff scattering

Figure 2 represents the Feynman diagrams for the Gravi-Compton-Primakoff scattering process $e^-(k_1)+\gamma(q_1)\to e^-(k_2)+G_{KK}(q_2)$ whose matrix element is

$$i\mathcal{M}_{\rm GCP} = \left(\frac{e\kappa}{2}\right)\overline{u}(k_2)\left[\frac{1}{s-m_e^2}\gamma_\mu k_{2\nu}(\not{k}+m_e)\gamma_\rho + \frac{1}{u-m_e^2}\gamma_\rho(\not{j}+m_e)\gamma_\mu k_{1\nu}\right.$$
$$\left. +\frac{2}{t}(-l\cdot q_1\gamma_\mu\eta_{\nu\rho}+\eta_{\mu\rho}\not{q}_1 l_\nu+\gamma_\mu l_\rho q_{1\nu}-\gamma_\rho l_\nu q_{1\mu})-\gamma_\mu\eta_{\nu\rho}\right]u(k_1)\epsilon^\rho(q_1)\epsilon^{\mu\nu *}(q_2), \quad (11)$$

where $s=k^2,\ t=l^2,\ u=j^2$ are the Mandelstam variables and $k=k_1+q_1$, $l=k_1-k_2$, $j=k_1-q_2$; $\epsilon^\rho(q_1)$ and $\epsilon^{\mu\nu}(q_2)$ are the polarization vector and tensor for the photon and KK graviton, respectively. The first two terms in Eq. (11) (Figs. 2(a) and 2(b)) represent Compton scattering and the third term (Fig. 2(c)) is the Primakoff process contribution; the last term (Fig. 2(d)) is due to the contact interaction. The Compton and Primakoff processes interfere and can not be separated; we therefore call this the Gravi-Compton-Primakoff (GCP) process.

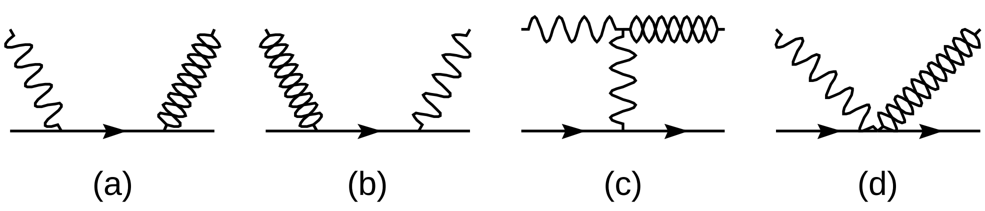


FIG. 2. Feynman diagrams for gravi-Compton-Primakoff scattering.

Since the electron mass is much larger than the temperature of the Sun and red giant cores where the GCP process is important, we calculate the cross section in the non-relativistic (NR) limit. We neglect plasma effects in our calculations.

In the NR limit, $T \ll m_e$, we neglect the initial electron momentum as well as the final-state electron recoil momentum; the final KK graviton therefore has the same energy as the incident photon energy, $\omega$. In the electron rest-frame, the Mandelstam variables have the following leading order approximation

$$s = m_e^2 + 2m_e\omega \ , \qquad t \simeq m_{\vec{n}}^2 - 2\omega^2 b_{\vec{n}} \ , \qquad u \simeq m_e^2 - 2m_e\omega + 2\omega^2 b_{\vec{n}} \ , \tag{12}$$

where $b_{\vec{n}} = 1 - \beta_{\vec{n}} \cos\theta_{\gamma\vec{n}}$, $\beta_{\vec{n}} = \sqrt{1 - x_{\vec{n}}}$, $x_{\vec{n}} = m_{\vec{n}}^2/\omega^2$, and $\theta_{\gamma\vec{n}}$ is the opening angle between the outgoing KK graviton and the incident photon. The matrix element squared is found to be (keeping only the leading term in $T/m_e$)

$$\sum{}' |\mathcal{M}_{\mathrm{GCP}}|^2 = \frac{1}{4} e^2 \kappa^2 m_e^2 f_{\mathrm{GCP}} \tag{13}$$

where

$$f_{\mathrm{GCP}} \simeq \frac{-4}{3(x_{\vec{n}} - 2b_{\vec{n}})^2} [4b_{\vec{n}}^4 - 2b_{\vec{n}}^3(7 + 2x_{\vec{n}}) + b_{\vec{n}}^2(18 + 15x_{\vec{n}} + x_{\vec{n}}^2) \\ -6b_{\vec{n}}(2 + 4x_{\vec{n}} + x_{\vec{n}}^2) + x_{\vec{n}}(6 + 9x_{\vec{n}} + x_{\vec{n}}^2)] \tag{14}$$

Neglecting the electron degeneracy, the volume emissivity is found to be

$$Q_{\mathrm{GCP}} \simeq n_e \int \frac{2d^3\vec{k}}{(2\pi)^3} \frac{\sum_{\vec{n}} \omega \sigma_{\mathrm{GCP}}(\omega, m_{\vec{n}})}{e^{\omega/T} - 1} \simeq \frac{\alpha n_e (4+n)! I_{\mathrm{GCP}}(n)}{2\pi m_e} \frac{T^{n+5}}{M_S^{n+2}} \tag{15}$$

where $\sigma_{\mathrm{GCP}}$ is the cross section for a single KK graviton, and the summation is over all kinematically accessible KK states with mass $m_{\vec{n}} \leq \omega$. The integral factor is

$$I_{\mathrm{GCP}}(n) = \int_0^1 dx_{\vec{n}} \ x_{\vec{n}}^{n/2-1} \int_{1-\beta_{\vec{n}}}^{1+\beta_{\vec{n}}} db_{\vec{n}} \ f_{\mathrm{GCP}} \ . \tag{16}$$

Numerically, the value of this integral is 12.0 (6.6) for $n = 2$ (3). For the red-giant core, electron degeneracy is relevant, but we expect this effect is of order unity, and the limits we derive using the non-degenerate formula should not be changed significantly.

### D. Gravi-bremsstrahlung

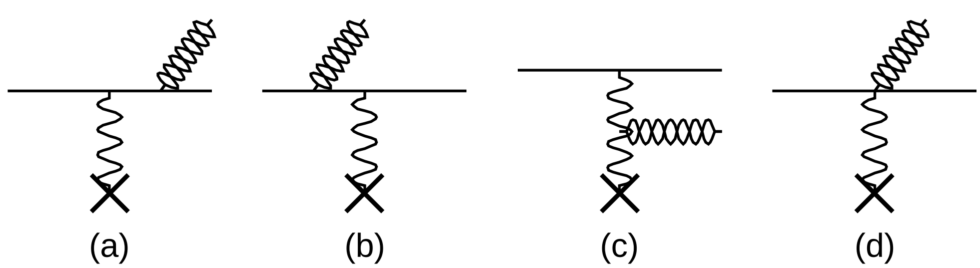


FIG. 3. Feynman diagrams for a bremsstrahlung emission of KK gravitons by electrons in the static electric field generated by nuclei.

Here we consider the bremsstrahlung emission of KK gravitons by electrons in the static electric field generated by nuclei. The diagrams are shown in Fig. 3, neglecting those with KK states emitted from the heavy nuclei, which are suppressed by the large nucleon mass. The $S$-matrix element for this process is similar to that of Eq. (11) and it reads

$$\begin{aligned}
\mathcal{S}_{\rm GB} &= i2\pi\delta(k_2^0+q_2^0-k_1^0)\mathcal{M}_{\rm GB}, \\
i\mathcal{M}_{\rm GB} &= \left(\frac{Ze^2\kappa}{2}\right)\overline{u}(k_2)\left[\frac{1}{s-m_e^2}\gamma_\mu k_{2\nu}(\not{k}+m_e)\gamma_0+\frac{1}{u-m_e^2}\gamma_0(\not{j}+m_e)\gamma_\mu k_{1\nu}+\right. \\
&\left.\frac{2}{t}(-l\cdot q_1\gamma_\mu\eta_{\nu 0}+\eta_{\mu 0}\not{q}_1 l_\nu+\gamma_\mu l_0 q_{1\nu}-\gamma_0 l_\nu q_{1\mu})-\gamma_\mu\eta_{\nu 0}\right]u(k_1)\left[\frac{1}{(\vec{k}_2-\vec{k}_1)^2+k_S^2}\right]\epsilon^{\mu\nu *}(q_2)\ , \qquad (17)
\end{aligned}$$

where $k_S$ in the denominator is a screening wave number, corresponding to the electrostatic potential of a point charge, $e^{-k_S r}/r$. The delta-function in the $S$-matrix element reflects the conservation of energy in the static electric field.

We calculate the energy-loss rate in the NR limit, where the initial and final state electrons have velocities $\vec{\beta}_i$ and $\vec{\beta}_f$, the virtual photon has momentum $m_e(\vec{\beta}_f-\vec{\beta}_i)$, and the KK graviton has energy $\omega_{\vec{n}}=\frac{1}{2}m_e(\beta_i^2-\beta_f^2)$. For the first two terms in Eq. (17), the leading and next-to-leading order terms in the velocity expansion cancel, so we need to retain all terms in the equation. The matrix element squared can be factorized as follows

$$\sum{}'|\mathcal{M}_{\rm GB}|^2 = \frac{Z^2e^4\kappa^2 f_{\rm GB}}{4m_e^2[{\beta_i}^2+{\beta_f}^2-2\beta_i\beta_f c_{if}+k_S^2/m_e^2]^2}\ , \qquad (18)$$

where $c_{if}=\cos\theta_{if}$ with $\theta_{if}$ the angle between the initial and final state electrons; and

$$\begin{aligned}
f_{\rm GB} \simeq & \frac{-11-26z^2-11z^4+48zc_{if}(1+z^2-zc_{if})}{3(1-z^2)^2} \\
&+\frac{32(1+z^2)^2-128zc_{if}(1+z^2-zc_{if})}{3(1+z^2-2zc_{if})^2}\ , \qquad (19)
\end{aligned}$$

where $z=\beta_f/\beta_i$. The volume emissivity is

$$\begin{aligned}
Q_{\rm GB} \simeq & \sum_j \frac{n_e n_j Z_j^2\alpha^2 m_e^{n+1}{\beta_i}^{2n+2}}{2^{n+1}M_S^{n+2}}\int_0^1 dx\ x^{n/2-1}\sqrt{1-x} \\
&\times\int_{-1}^1 dc_{if}\int_0^1 dz\ z^2(1-z^2)^{n+2}\frac{y_{\rm GB}}{[1+z^2-2zc_{if}+k_S^2/m_e^2{\beta_i}^2]^2}\ , \qquad (20)
\end{aligned}$$

where the summation of $j$ is over all species of nuclei in the star. Neglecting $k_S$ and averaging $\beta_i$ over a Maxwell-Boltzmann velocity distribution, we have

$$Q_{\rm GB} \simeq \sum_j \frac{\Gamma(\frac{5}{2}+n)n_e n_j Z_j^2\alpha^2 I_{\rm GB}(n)}{\Gamma(\frac{3}{2})}\frac{T^{n+1}}{M_S^{n+2}}\ . \qquad (21)$$

The numerical value of the integral $I_{\rm GB}(n)$ is 0.7 (0.3) for $n=2$ (3).

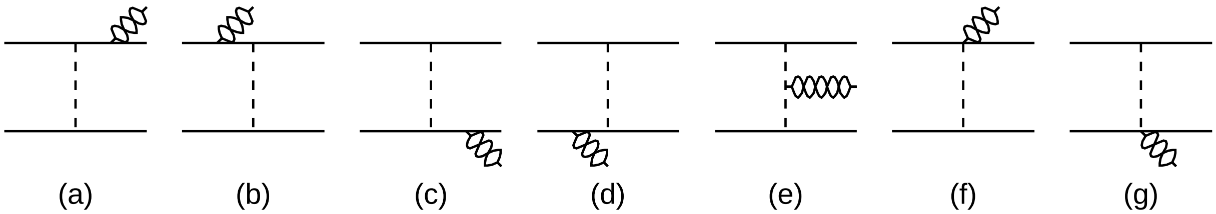


FIG. 4. Representative Feynman diagrams for a nucleon-nucleon bremsstrahlung emission of KK gravitons. The contact interaction diagrams (f) and (g) are zero since the KK graviton is on-shell.

### E. Nucleon-nucleon bremsstrahlung

For the nucleon-nucleon bremsstrahlung process as shown in Fig. 4, we take the standard pion-nucleon Yukawa interaction

$$\mathcal{L} \;=\; -ig_{\pi NN}\bar{N}\vec{\tau}\cdot\vec{\pi}\gamma_5 N \;, \tag{22}$$

where $g_{\pi NN} \simeq 13.5$ is the pion-nucleon coupling and the $\tau_i$ are the Pauli matrices. The contact interaction of a KK graviton with a nucleon and pion can be derived from this Lagrangian. The Feynman rules are

$$\begin{array}{ll} \pm g_{\pi NN}\gamma_5\eta_{\mu\nu} & \text{for } nn\pi^0 h^{\vec{n}}_{\mu\nu} \text{ and } pp\pi^0 h^{\vec{n}}_{\mu\nu}, \\ \sqrt{2}g_{\pi NN}\gamma_5\eta_{\mu\nu} & \text{for the } np\pi^{\pm} h^{\vec{n}}_{\mu\nu}. \end{array}$$

The other relevant Feynman rules can be found in Ref. [6].

It is most convenient to carry out the calculation in the center-of-mass frame. The initial and final states both have center-of-mass momentum $\vec{P}$, then all other momenta can be written in terms of $\vec{P}$ and the relative momenta, $\vec{p}_{1,2} = \vec{P} \pm \vec{p}$ and $\vec{p}_{3,4} = \vec{P} \pm \vec{q}$. There are in total 14 diagrams contributing to the nucleon-nucleon bremsstrahlung process, as shown in Fig. 4 plus other 7 fermion interchange diagrams. The four contact interaction diagrams (Fig. 4(f,g) and their interchange diagrams) are automatically zero, because the KK gravitons are on-shell. The other 10 diagrams can be grouped into two sets, $A$ and $B$, with set $A$ the exchange diagrams of set $B$. The matrix element-squared can be factorized as (neglecting the pion mass since $m_\pi^2 \ll m_N T_{\rm SN}$ in the supernova core)

$$\sum{}' |\mathcal{M}_{\rm NB}|^2 \;=\; \frac{1}{4} g_{\pi NN}^4 \kappa^2 f_{NN} \;, \tag{23}$$

where in the one-pion exchange approximation

$$f_{NN} = \begin{cases} \frac{1}{4}\left(|\mathcal{M}_A|^2 + |\mathcal{M}_B|^2 - 2|\mathcal{M}_A\mathcal{M}_B|\right) & \text{for } nn \text{ or } pp, \\ |\mathcal{M}_A|^2 + 4|\mathcal{M}_B|^2 + 4|\mathcal{M}_A\mathcal{M}_B| & \text{for } np, \end{cases}$$

with (keeping only the leading term in $T_{\rm SN}/m_N$)

$$|\mathcal{M}_A|^2 \;=\; |\mathcal{M}_B|^2 \;\simeq\; 7 + 9r_{\vec{n}} \;, \qquad |\mathcal{M}_A\mathcal{M}_B| \;\simeq\; 4 + 5r_{\vec{n}} \tag{24}$$

and $r_{\vec{n}} = m_{\vec{n}}^2/E_{\vec{n}}^2$.

The volume emissivity through the nucleon-nucleon bremsstrahlung process has a relatively simple expression in the non-degenerate limit in which the final-state Pauli blocking ($1 - f_{\rm FD} \simeq 1$) is neglected. Defining $z_{\vec{n}} = E_{\vec{n}}/T$, we obtain

$$Q_{\rm NB} \simeq \frac{g_{\pi NN}^4 n_N^2 I_{\rm NB}(n)}{64\pi^{5/2} m_N^{5/2}} \frac{T^{n+7/2}}{M_S^{n+2}} \tag{25}$$

where

$$I_{\rm NB}(n) \simeq \int_0^\infty dz_{\vec{n}}\, e^{-z_{\vec{n}}} z_{\vec{n}}^{n+2} (1 + \frac{\pi}{4} z_{\vec{n}})^{1/2} \int_0^1 dr_{\vec{n}}\, r_{\vec{n}}^{n/2-1} (1 - r_{\vec{n}})^{1/2} f_{NN} \ . \tag{26}$$

Numerically, the value of this integral is about 80 ($n = 2$) and 280 ($n = 3$) for $nn$ and $pp$ bremsstrahlung, and 2700 ($n = 2$) and 9300 ($n = 3$) for $np$ bremsstrahlung.

## III. LIMITS ON $M_S$

We apply our formulae in the previous section to the energy losses of the Sun, red giants and SN1987A. We briefly review the arguments for these cases as follows [12]:

(a) Our Sun is a thermal system with a temperature $1.55 \times 10^7$ K $= 1.3$ keV, where thermal pressure balances gravity. If the Sun excessively losses energy to the KK gravitons, its radius will shrink and the temperature rise. The Sun would then need to burn more nuclear fuel to compensate for the decrease of gravitational energy. This might result in a solar age shorter than the current value $4.5 \times 10^9$ yr. To avoid too rapid consumption of the nuclear fuel, a conservative requirement is that the energy-loss rates from the KK processes do not exceed the solar luminosity, $\mathcal{L}_\odot = 3.90 \times 10^{33}$ erg sec$^{-1}$.

(b) If the core of a red giant near the helium flash ($T \sim 8.6$ keV) produces excessive KK gravitons, then the helium core may not be ignited and the star would become a helium white dwarf after ascending the red-giant branch, contrary to the observation of horizontal branch stars. This requires that the KK emission to be less than the red-giant luminosity at helium flash, $\sim 2000\ \mathcal{L}_\odot$.

(c) Observational data on SN1987A from IMB and Kamiokande experiments imply $E \geq 2 \times 10^{53}$ ergs emitted over a diffusion period of the order of 10 seconds in form of neutrino flux. This means that much of the binding energy of a neutron star, $\sim 3 \times 10^{53}$ ergs, is carried away by neutrinos; therefore the energy-loss rate from KK states should be less than $\sim 10^{52}$ erg sec$^{-1}$.

Finally, we note that since the temperatures for the Sun and the red-giant core are fairly low, only KK gravitons for the cases of $n = 2$ and 3 extra dimensions can be efficiently produced there. In the following we only consider the limits for $n = 2$ and 3.

For the Sun and the red-giant core, we need to consider photon-photon annihilation, GCP scattering and gravi-bremsstrahlung processes. The calculated energy loss rates per unit mass for these three processes are presented in Table I(a), scaled with $M_S$ in units of

| (a) | Sun | | Red giants | |
|---|---|---|---|---|
| | $n=2$ $(\times M_S^{-4})$ | $n=3$ $(\times M_S^{-5})$ | $n=2$ $(\times M_S^{-4})$ | $n=3$ $(\times M_S^{-5})$ |
| $\dot{\varepsilon}_\gamma$ | $1.7\times10^{-3}$ | $1.6\times10^{-11}$ | 6.3 | $4.0\times10^{-7}$ |
| $\dot{\varepsilon}_{\rm GCP}$ | $1.3\times10^{-4}$ | $6.2\times10^{-13}$ | 50 | $1.7\times10^{-6}$ |
| $\dot{\varepsilon}_{\rm GB}$ | $7.6\times10^{-4}$ | $1.9\times10^{-12}$ | $10^3$ | $1.7\times10^{-5}$ |

| (b) | SN1987A $n=2$ $(\times M_S^{-4})$ | $n=3$ $(\times M_S^{-5})$ |
|---|---|---|
| $\dot{\varepsilon}_\gamma$ | $4.7\times10^{23}\ T_{30}^{9}$ | $1.1\times10^{20}\ T_{30}^{10}$ |
| $\dot{\varepsilon}_e$ | $8.8\times10^{17}$ $1.9\times10^{21}$ $1.9\times10^{25}$ | $2.3\times10^{14}$ $6.0\times10^{17}$ $9.8\times10^{21}$ |
| $\dot{\varepsilon}_{\rm NB}$ | $6.7\times10^{25}\ T_{30}^{11/2}$ | $6.3\times10^{21}\ T_{30}^{13/2}$ |

TABLE I. Energy loss rates (in units of erg g$^{-1}$sec$^{-1}$) due to escaping KK gravitons (a) for the Sun and a red giant from photon-photon annihilation ($\dot{\varepsilon}_\gamma$), Gravi-Compton-Primakoff scattering ($\dot{\varepsilon}_{\rm GCP}$) and Gravi-bremsstrahlung ($\dot{\varepsilon}_{\rm GB}$); and (b) for a supernova from photon-photon annihilation ($\dot{\varepsilon}_\gamma$), electron-positron annihilation ($\dot{\varepsilon}_e$) and nucleon-nucleon bremsstrahlung ($\dot{\varepsilon}_{\rm NB}$). The scaling with $M_S$ (in units of TeV) has been factored out. The three numbers for $\dot{\varepsilon}_e$ correspond to the supernova temperature $T_{\rm SN}=20,30,60$ MeV. $T_{30}\equiv T_{\rm SN}/30$ MeV.

TeV. We have used the electron densities [2] $n_e\simeq 6.3\times10^{25}$ cm$^{-3}$ and $3.0\times10^{29}$ cm$^{-3}$, and the mass densities 156 g cm$^{-3}$ and $10^6$ g cm$^{-3}$ for the Sun and the red-giant core near helium flash respectively. For the case of supernovae, we consider the photon-photon annihilation, electron-positron annihilation and nucleon-nucleon bremsstrahlung processes; those results are shown in Table I(b). We take the supernova core density $\simeq 10^{15}$ g cm$^{-3}$ and neutron fraction to be 1.

Using the conservative upper limits on the energy-loss rates of

$$\dot{\varepsilon}_{\rm Sun}\sim 1 \text{ erg g}^{-1}\text{sec}^{-1},\quad \dot{\varepsilon}_{\rm RG}\sim 100 \text{ erg g}^{-1}\text{sec}^{-1}\quad\text{and}\quad \dot{\varepsilon}_{\rm SN}\sim 10^{19}\text{ erg g}^{-1}\text{sec}^{-1}, \tag{27}$$

we obtain the lower limits on $M_S$ summarized in Table II.

| $n$ | Sun | Red giant | SN1987A |
|---|---|---|---|
| 2 | $0.20^{(a)}$, $0.11^{(c)}$, $0.17^{(d)}$ | $0.50^{(a)}$, $0.84^{(c)}$, $1.8^{(d)}$ | $15\ T_{30}^{2.25(a)}$, $(0.5-37)^{(b)}$, $51T_{30}^{1.375(e)}$, $(30-130)^{sum}$ |
| 3 | | | $1.6\ T_{30}^{2(a)}$, $(0.1-4.0)^{(b)}$, $3.6T_{30}^{1.3\ (e)}$, $(2.1-9.3)^{sum}$ |

TABLE II. Limits to $M_S$ in TeV from (a) photon-photon annihilation, (b) electron-positron annihilation, (c) gravi-Compton-Primakoff scattering, (d) gravi-bremsstrahlung and (e) nucleon-nucleon bremsstrahlung. The numbers in brackets correspond to the supernova temperature range $T_{\rm SN}=20-60$ MeV. For "sum", all contributing processes (a,b,e) are included. $T_{30}\equiv T_{\rm SN}/30$ MeV.

[2] All parameters are taken from Ref. [12].

## IV. DISCUSSION AND CONCLUSION

We have calculated the energy-loss rates for the Sun, red giants and supernovae due to the emission of KK gravitons. The lower limits on the string scale $M_S$ can be derived by requiring the energy-loss rates to be smaller than the respective observed luminosities of those stars. We found the lower limits from the Sun and red-giants are in the range of several hundred GeV with two large extra dimensions. The lower limits from the supernova SN1987A are more stringent, in particular the nucleon-nucleon bremsstrahlung process gives

$$M_S \gtrsim \begin{cases} 51\ (T_{\rm SN}/30\ {\rm MeV})^{11/8}\ \ {\rm TeV} & \text{for } n=2, \\ 3.6\ (T_{\rm SN}/30\ {\rm MeV})^{13/10}\ \ {\rm TeV} & \text{for } n=3. \end{cases}$$

Our supernova result is consistent with that of Ref. [11].

The most important, yet uncertain, parameter in our analyses is the supernova temperature. We see from Table II that our final results on the $M_S$ limit, including all contributing processes in supernovae, range from $30-130$ TeV ($2.1-9.3$ TeV) for $n$=2 (3), corresponding to $T_{\rm SN}=20-60$ MeV. Although there are other sources of uncertainties in our calculations, such as in astrophysical parameters, corrections due to plasma effects, electron degeneracy, screening effects etc., we do not expect these to significantly alter the $M_S$ limit. because of the high-power $M_S^{-(n+2)}$ dependence of the emission rate. We note that there also exists an interesting lower bound from the consideration of the decay of relic graviton KK states to photons, which can distort the diffuse photon spectrum [15]. That limit is at the order of 110 TeV for $n=2$, but it has uncertainties associated with cosmological models.

The lower limits obtained from the astrophysical processes can be complementary to those from collider experiments [6–8], in particular to the collider processes with virtual KK state exchange, which has the string scale dependence of $M_S^{-4}$, essentially independent of $n$. On the other hand, a string scale at the order of 50 TeV for $n=2$ would make string effects inaccessible at collider experiments. From Eq. (3), this scale corresponds to two compact dimensions of the size of about $10^{-4}$ mm, which is beyond the sensitivity of the tabletop gravitation experiments being planned [5]. Finally, from theoretical point of view, such a high scale for two extra dimensions makes the KK scenario less attractive since one of the motivations for introducing a TeV-scale string theory is to solve the weak-GUT scale hierarchy problem.

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